Salt Lake Community College
Math 1060 Final Exam - Fall Semester 2009

Name:_____

Part II – Work all of the following problems. Be sure to show your work where appropriate. Unsupported answers may not receive full credit. You will need a calculator for some problems. When directions specify "exact value" a calculator should **not** be used.

When simplifying answers, it is **not** necessary to rationalize denominators.

Students are NOT allowed to use books or notes.

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10)
a) Use a calculator to find the value of the expression in radians rounded to the nearest hundredth.

$\csc^{-1}\left(\frac{5}{3}\right) =$ .64 rad

b) Find the **exact** value of the expression.

$\cot(\sin^{-1}(\frac{\sqrt{3}}{2})) =$ $\cot\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{\sqrt{3}}}$ or $\frac{\sqrt{3}}{3}$

11) Find **all real numbers** that satisfy the equation. Please give **exact** answers, not decimal approximations.

$\sqrt{2}\cos(3x)+1=0$

$\cos(3x) = -\frac{1}{\sqrt{2}}$

$3x = \frac{3\pi}{4} + 2k\pi$

$\boxed{x = \frac{\pi}{4} + \frac{2k\pi}{3}}$

OR

$3x = \frac{5\pi}{4} + 2k\pi$

$\boxed{x = \frac{5\pi}{12} + \frac{2k\pi}{3}}$

12) The second hand of a clock is 5 inches long.

a) How far does the tip of the second hand move in 15 seconds ? [Hint: This distance corresponds to an arc length.]

$S = \alpha r$

$S = \left(\frac{\pi}{2}\right)(5) = \frac{5\pi}{2}$ in

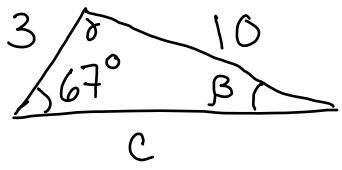
b) What is the angular velocity of the second hand in radians per second?

$\omega = \frac{\alpha}{t}$ $\qquad \omega = \frac{2\pi \text{ rad}}{60 \text{ sec}}$

$\omega = \frac{\pi}{30} \frac{\text{rad}}{\text{sec}}$

13) Two sides and an angle are given. Solve any triangle(s) that results. Round to the nearest tenth if necessary.

a = 10, b = 3, $\alpha = 67^\circ$



$\frac{\sin 67^\circ}{10} = \frac{\sin \beta}{3}$

$\frac{3 \sin 67^\circ}{10} = \sin \beta$

$\beta \approx 16.0^\circ$

$180^\circ - 16^\circ = 164^\circ$ impossible (only 1 triangle)

$\gamma = 180^\circ - (67^\circ + 16^\circ)$

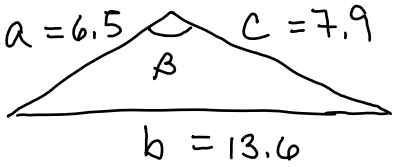
$\gamma = 97^\circ$

$\frac{c}{\sin 97^\circ} = \frac{10}{\sin 67^\circ}$

$c = \frac{10 \sin 97^\circ}{\sin 67^\circ} \approx 10.8$

$c \approx 10.8$

14) Find the measure of the angle β in degrees . Round to the nearest tenth.



$(13.6)^2 = (6.5)^2 + (7.9)^2 - 2(6.5)(7.9)\cos\beta$

$184.96 = 104.66 - 102.7\cos\beta$

$80.3 = -102.7\cos\beta$

$-\frac{80.3}{102.7} = \cos\beta$

$\beta = 141.4^\circ$

15) Find the following for the vectors $\mathbf{v} = \langle -3, 2 \rangle$ and $\mathbf{w} = \langle 4, 1 \rangle$

a) $3\mathbf{v} - 4\mathbf{w}$ $= \langle -9, 6 \rangle - \langle 16, 4 \rangle = \langle -25, 2 \rangle$

b) $|\mathbf{v}| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13}$

c) $|\mathbf{w}| = \sqrt{(4)^2 + (1)^2} = \sqrt{17}$

d) $\mathbf{v} \cdot \mathbf{w} = (-3)(4) + (2)(1) = -10$

e) The smallest positive angle between $\mathbf{v}$ and $\mathbf{w}$. Round your answer to the nearest tenth of a degree.

$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|v\|\|w\|} = \frac{-10}{\sqrt{13}\sqrt{17}}$

$\theta = 132.3^\circ$

16) Perform the indicated operation and **write the answer in the form a + bi**. Give **exact** values, not decimal approximations.

$4\left(\cos(\frac{\pi}{3})+i\sin(\frac{\pi}{3})\right)\cdot 3\left(\cos(\frac{\pi}{6})+i\sin(\frac{\pi}{6})\right)$

$= (4\cdot 3)\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right]$

$= 12\left[\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right]$

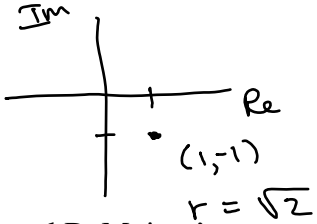
$= 12(0+i(1))$

$= \boxed{12i}$

17)

a) Convert the complex number to trigonometric form with $0^\circ \le \theta < 360^\circ$

$1-i = \sqrt{2}\left(\cos(315^\circ)+i\sin(315^\circ)\right)$



b) Simplify the following by using the trigonometric form from part (a) and DeMoivre's Theorem. **Leave the final answer in trigonometric form,** $0^\circ \le \theta < 360^\circ$ **.**

$(1-i)^5 = \left(\sqrt{2}\right)^5\left(\cos[5(315^\circ)]+i\sin[5(315^\circ)]\right)$

$= 4\sqrt{2}\left(\cos(1575^\circ)+i\sin(1575^\circ)\right)$

$= \boxed{4\sqrt{2}\left(\cos(135^\circ)+i\sin(135^\circ)\right)}$

18) Write an equivalent rectangular equation for the given polar equation.

$r = 3\sin\theta$

$r^2 = 3r\sin\theta$

$x^2 + y^2 = 3y$

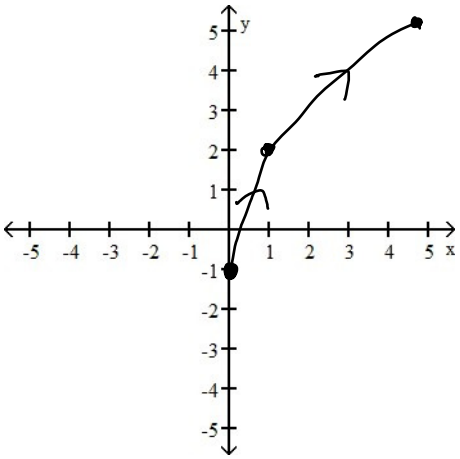
19) Graph the curve whose parametric equations are given. List at least three points on the curve. Indicate the orientation of the graph.

$x = t^2$

$y = 3t - 1$

$0 \le t \le 2$

t	x	y
0	0	-1
1	1	2
2	4	5



Part III – Work 3 of the problems in this section. Cross out any problems you do not want graded.
Be sure to show your work where appropriate. Unsupported answers may not receive full credit. You will need a calculator for some problems. When directions specify "exact value" a calculator should **not** be used.

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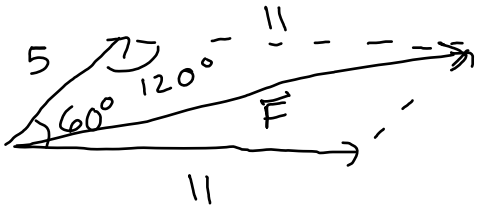
20) Forces of 11 lb and 5 lb act at an angle of 60° to each other. Find the magnitude of the resultant force. Round your answer to the nearest tenth.

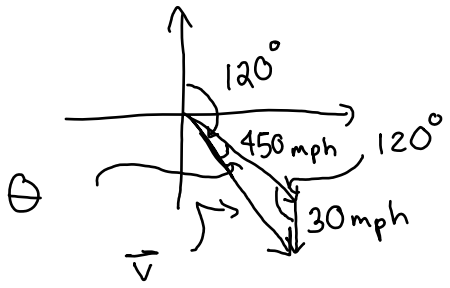


$$|\vec{F}|^2 = 11^2 + 5^2 - 2(5)(11)\cos(120^\circ)$$

$$|F|^2 = 201$$

$$|F| = \sqrt{201} \approx \boxed{14.2 \text{ lb}}$$

21) An airplane is heading on a bearing of 120° with an airspeed of 450 mph. The wind is blowing from the north (bearing 180°) at 30 mph. Find the ground speed of the airplane and the bearing of the course. Round your answers to the nearest tenth.



$$|\vec{v}|^2 = 450^2 + 30^2 - 2(450)(30)\cos(120^\circ)$$

$$|\vec{v}|^2 = 216{,}900$$

Ground speed:

$$|\vec{v}| = \sqrt{216{,}900} \approx \boxed{465.7 \text{ mph}}$$

$$\frac{\sin\theta}{30} = \frac{\sin 120^\circ}{465.7}$$

$$\sin\theta = \frac{30\sin 120^\circ}{465.7}$$

$$\theta \approx 3.2^\circ$$

$$\boxed{\text{Bearing: } 123.2^\circ}$$

22) Write a set of parametric equations that will produce the indicated graph.

The line segment starting at (0,1) with t = 0, and ending at (3,-2) with t = 1.

x = $3t$

y = $-3t + 1$

$0 \le t \le 1$

$x = mt + b \Rightarrow x = 3t$

t	x
0	0
1	3

$y = mt + b \Rightarrow y = -3t + 1$

t	y
0	1
1	-2

23) Prove that the following equation is an identity.

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \tan\alpha + \tan\beta$$

24) Find all real numbers in the interval $[0, 2\pi)$ that satisfy the equation.

$\sin^2\theta + \sin\theta = \cos^2\theta$

$\sin^2\theta + \sin\theta = 1 - \sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\theta = \frac{3\pi}{2}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

25) Complete the table and graph the polar equation.

θ	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	2π
r	2	1	$1-\frac{\sqrt{2}}{2}$	0	$1-\frac{\sqrt{2}}{2}$	1	2
			$\approx .3$		$\approx .3$		

$r = 1 + \cos\theta$